Motivating Diffeomorphism Groups

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February 11, 2026

[Hat], [Kup], [Bud]. Our project is essentially to try and understand what we can about automorphism groups of smooth manifolds, and in particular the sort of initial such object $\mathrm{Diff}_\partial(D^n)$. I am attracted to this issue because for once it doesnt seem to bear much expalining. But regardless here are some reasons.

- "Automorphisms are always interesting!" - Hatcher.
- Smooth bundles with fiber M are classified by homotopy classes of maps into $B\mathrm{Diff}(M)$.
- There is a highly non-obvious connection with algebraic K theory. Notice that one needent care about either independently to recognise that this connection itself is interesting.
- (Kupers) They give a precise statment of differences between the smooth, PL and topological manifold categories, through statements like Alexanders trick.
- They give precise statments for the concept in manifold theory that even though everything is locally trivial there can be global non-trivial structures.
- (Budney) Many theorems in algebraic topology / manifold topology can be stated in terms of diffeomorphisms. He gives the example of linking number and Schonenflies "basically" being the statement that $\mathrm{Diff}_\partial(S^1 \times D^1) \cong \mathbb{Z}$. Alexanders Theorem and Dehns lemma can be stated as $\mathrm{Diff}_\partial(S^1 \times D^2) \cong *$.
- The group of diffeomorphisms has transitive actions on many other spaces in geometric topology, for instance the space of embeddings of the disc etc. Which allows you to exihibit them as homogeneous spaces.
- The connection with smooth structures on spheres, homotopy spheres and the stable stems provided by Kervaire-Milnor.
- It is one of the few big open problems left in the sense that not a single one is known (dim geq 5).
- "A promising future" Recent work has begun to open up the field.

References

[Bud] Ryan Budney. Topology in Dimension 4.5 – Session C Motivation and Background.

[Hat] Allen Hatcher. A 50 -Year View of Diffeomorphism Groups.

[Kup] Alexander Kupers. Lectures on diffeomorphism groups of manifolds, version February 22, 2019.